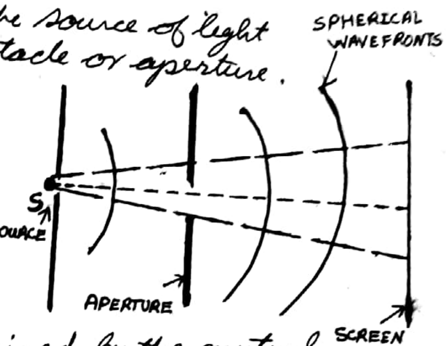
The phenomenon of bending of light round corners and spreading of light waves into the geometrical shadow of an object is called as diffraction.

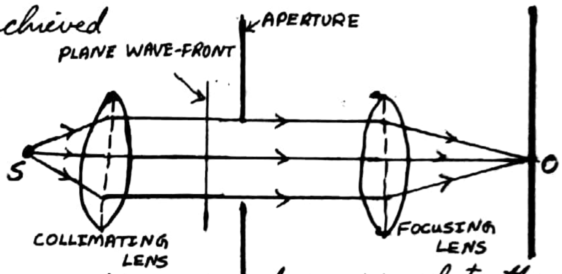
This phenomenon is more pronounced when the size of the aperture is comparable to the wavelength of light.

There are two types of diffractions i) FRESNEL'S and ii) FRAUNHOFER.

i) FRESNEL'S DIFFRACTION :- In this type of diffraction the source of light or screen or both are at finite distances from the obstacle or aperture. No lenses are used to make the light parallel or convergent. The incident wave-front is not plane but is either spherical or cylindrical. Thus the phase of secondary wavelets is not the same at all points in the plane of the aperture or the obstacle causing diffraction i.e the resultant amplitude at any point of the screen is obtained by the mutual interference of secondary wavelets from different elements of unblocked portions of wave front. eg St edge; narrow slit; a thin wire; a small circular hole; → Applied in the construction of a zone plate.

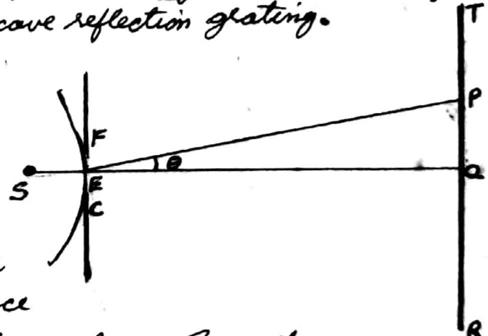


ii) FRAUNHOFER'S DIFFRACTION :- In this type of diffraction the source of light and the screen are effectively at infinite distances from the aperture or obstacle which causes diffraction. This may be achieved by using two convex lenses, one to make the light from the source parallel before it falls on the aperture and the other to focus the light after diffraction on the screen. This arrangement in fact removes the sources and the screen to infinity. The incident wave-front is plane and the secondary wavelets thus originate from the unblocked portions of the wave-front are in the same phase at every point in the plane of the aperture. Thus the diffraction is produced by the interference between parallel rays which are brought into focus with the help of a convex lens. eg Plane transmission grating concave reflection grating.



EXPLAINATION

Fresnel made use of Huygen's Principle in explaining the phenomenon of Diffraction. According to it each of exposed wavefront in the aperture acts as a source of secondary waves which interfer with each other to produce diffraction. The exposed part is assumed to be made up of parts or elements called as Fresnel's Zones. The resultant on the screen depends upon the combined effect of the secondary waves originating from these various zones. Moreover the effect at a point on screen, due to a zone depends upon its distance from the zone. As shown in figure the effect at pt. P due to wavefront at E depends upon $(1+\cos\theta)$, also called as oblique factor here $\theta = \angle PEO$.



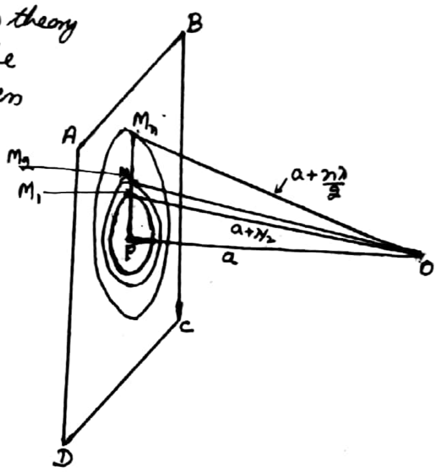
As per figure. i) Effect of E wavefront at pt. O is maximum as $\theta=0 \Rightarrow \cos\theta=1$

ii) As we move towards P, θ increases (or $\cos\theta$ decreases) thus effect decreases.

iii) In direction EF the effect is half that at O as $\theta=90^\circ$ & $\cos\theta=0$

iv) In direction ES (towards source) the resultant effect is zero as $\theta=180^\circ$ or $\cos\theta=-1$ (Non existance of wavefronts in the backward direction)

<u>HALF-PERIOD ZONES</u> Let ABCD be a plane wave front at a distance 'a' from the source O. As per Huygen's theory every point on the wavefront is regarded as the origin of secondary wavelets and at a given instant every one of these secondary wavelets pass through the pt. O. Thus resultant effect at O is the combination of all these wavelets. To determine it we divide the entire wave front into concentric zones as follow.



From O drop a ⊥ on ABCD (at pt P). This pt is called as a "Pole" of OP = a and λ = wavelength.
Now O as centre and radius $(a+\lambda/2)$ draw a sphere cutting the wave front in a circle M_1, then

$OM_1 = a+\lambda/2$ and $OM_1 - OP = \lambda/2$. i.e. secondary wavelets originating from pt P. and any pt. on circle M_1 have a phase difference given by

$$\frac{2\pi}{\lambda}(OM_1 - OP) = \pi$$

Since π radians ≡ phase difference T/2 thus area enclosed by circle M_1 is called as Fresnel's first half period zone.

Similarly we construct other spheres of radii $(a+2\lambda/2)$; $(a+3\lambda/2)$; $(a+4\lambda/2)$ .... $(a+\frac{n\lambda}{2})$ intersecting the wavefront ABCD in circles M_2; M_3; M_4 .... M_n

These zones are the annular rings, called as half period zones as the waves reaching O from P and M_1 differ by phase π or T/2. and waves reaching O from M_1 and M_2 also differ in phase by π or T/2.

<u>Rectilinear Propagation of light</u> To explain it we have to find the resultant effect of the whole wavefront at the pt. O. To do it we divide the wavefront into half period zones. Thus we have to find the resultant of the wavefronts from these zones at pt. O.

Now radius of 1st half period zone $PM_1^2 = (a+\lambda/2)^2 - a^2 = (a^2 + \frac{\lambda^2}{4} + a\lambda - a^2) = a\lambda$
(neglecting $\lambda^2/4$).

∴ Area of 1st half period zone = $\pi a\lambda$. —— (1)

For nth half period zone. $PM_n^2 = (a+n\lambda/2)^2 - a^2 = an\lambda$.
for (n-1)th " " " $PM_{n-1}^2 = (a+(n-1)\lambda/2)^2 - a^2 = a(n-1)\lambda$.

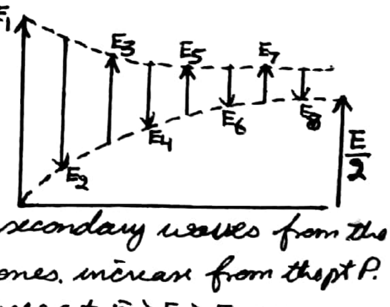
∴ Area of nth zone $= \pi(PM_n^2 - PM_{n-1}^2) = \pi an\lambda - \lambda a(n-1)\lambda = \pi a\lambda$ —— (2)

clearly it is same as that of the 1st half period zone.

Thus the areas of the various half period zones are independent of the order of the zone and are nearly equal. The radii of these zones being ~~equal~~ proportional to $\sqrt{n}$ where n = 1, 2, 3 .... etc.

<u>Net Intensity of light</u>

Now the net intensity of light due to each zone at the pt. O depends upon i) Distance of O from wavefront ii) Area of zone and iii) oblique factor.



Let $E_1, E_2, E_3 \ldots E_n$ represent the amplitudes of secondary waves from the various half period zones. As the distances of the zones increase from the pt P. the oblique factor increases i.e. amplitude decreases s.t. $E_1 > E_2 > E_3 \ldots$
The resultant amplitude at O is

$$E = E_1 - E_2 + E_3 - E_4 + \ldots + E_n \text{ —— (3)}$$

Now $E_2 = \frac{E_1+E_3}{2}$; $E_4 = \frac{E_3+E_5}{2}$ —— (4) substituting it we get. (for odd n)

$$E = \frac{E_1}{2} - \ldots \text{; } E = E_1 - \left(\frac{E_1+E_3}{2}\right) + E_3 - \left(\frac{E_3+E_5}{2}\right) + \ldots + E_n/2 = \frac{E_1}{2} + \frac{E_n}{2} \text{ —— (4)}$$

Thus resultant amplitude at O is clearly half the sum of the amplitudes contributed by the first and the last zones. If n is sufficiently large the effect due to the last zone becomes negligible and the amplitude due to the whole wave $= E_1/2$.

Also as intensity is proportional to square of amplitude $\therefore I = \frac{E_1^2}{4}$

i.e the intensity at O due to the waves from all the zones is equal to "One-fourth" of the intensity due to the waves from the first half period zone.

Now if an obstacle is placed at P, the resultant disturbance at O. is equal to half the ~~d~~ disturbance due to the first exposed zone. As the displacement decreases rapidly as the order of the zone, if the obstacle at P blocks a considerable number of half period zones the effect is ~~negligible~~ pronounced and practically no light will be recieved at O. In other words. light travels approx. in a straight line.

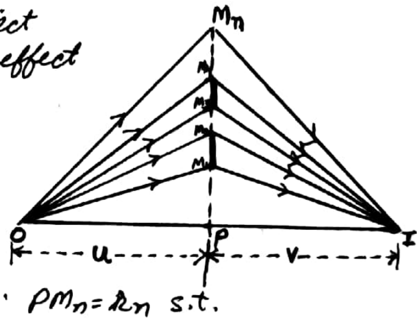
Zone Plate

It is a transparent plate on which circles whose radii are proportional to the square roots of natural numbers 1, 2, 3.... are drawn. The alternate zones thus formed are blocked. Such a plate behaves like a convex lens and produces an image of a source of light on the screen placed at a suitable distance

Construction We draw concentric circles on white paper s.t their radii are proportional to the square root of the natural numbers. The odd being covered with black ink and a reduced photograph is taken. The negative appears as shown in fig (a) while developed photograph is as shown in fig(b). Thus alternate zones cut off light.

(a) (b)

Theory or Action Let O be a luminous point object emitting spherical waves of wavelength λ whose effect at the pt. I on the screen is required.
Consider an imaginary plane of transparent medium be present at P lying $\perp$ to the plane of the paper and joining OI. Divide this plane into zones bounded by circles having centres at P and radii $PM_1 = r_1$; $PM_2 = r_2 \cdots PM_n = r_n$ s.t.



$$OM_1 + IM_1 = OP + IP + \lambda/2.$$
$$OM_2 + IM_2 = OP + IP + \frac{2\lambda}{2}.$$
$$\vdots$$
$$OM_n + IM_n = OP + IP + \frac{n\lambda}{2}.$$

These rings are the half period zones. for pt I as the path difference of any two consecutive zones, differs by $\lambda/2$.

I) Radius of nth circle

$$OM_n + IM_n = OP + IP + \frac{n\lambda}{2} \quad \text{——— (1)}$$

Let $OP = u$; $IP = v$.

Now.

$$OM_n = (OP^2 + PM_n^2)^{1/2} = (u^2 + r_n^2)^{1/2}$$
$$= u\left(1 + \frac{r_n^2}{u^2}\right)^{1/2} = u + \frac{r_n^2}{2u}.$$

(using Binomial Th. & neglecting higher terms)

Similarly $IM_n = (v^2 + r_n^2)^{1/2} = v + \frac{r_n^2}{2v}$

Substituting these values in (1) we get

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + n\lambda/2$$
$$r_n^2\left\{\frac{1}{u} + \frac{1}{v}\right\} = n\lambda$$
$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2} \quad \text{——— (2)}$$
$$\text{or } r_n^2 = \frac{n\lambda uv}{u+v} \quad \text{——— (3)}$$

as u, v and λ are constants thus

$$r_n \propto \sqrt{n} \quad \text{——— (4)}$$

Also the area of the nth zone is given by

$$\pi(r_n^2 - r_{n-1}^2) = \pi\left\{\frac{n\lambda \cdot uv}{u+v} - \frac{(n-1)\lambda uv}{u+v}\right\} = \frac{\pi\lambda uv}{u+v}.$$

This is clearly independent of n, hence for a given object and image the areas of all the zones remains the same.

If now $E_1, E_2, E_3, \ldots E_n$ be the amplitudes of light from Ist, IInd ... nth zone. Then intensity of light at pt I is

$$E = E_1 - E_2 + E_3 - E_4 + \cdots + E_n. \quad \text{— (5)}$$

$$E = E_1 - \left(\frac{E_1+E_3}{2}\right) + E_3 - \left(\frac{E_3+E_5}{2}\right) + \cdots \frac{E_n}{2} = \frac{E_1}{2} + \frac{E_n}{2} \ (\text{for odd } n)$$

& $E = \frac{E_1}{2}$ is n is very large & odd.

If the even number of zones are painted black and do not allow the transmission of light. Then intensity of light at I is

$$E' = E_1 + E_2 + E_3 + \cdots \quad \text{— (6)}$$

clearly $E' > E$ i.e the intensity (proportional to E^2) due to the presence of zone plate is much heigher than in its absence or that the whole of the primary wavefront is unobstructed

<u>Focal Length</u>

As per relation eqn (2) we have.

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2}$$ this is similar to the one found for convex lens

i.e $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Thus the focal length of the zone plate is $f_n = \frac{r_n^2}{n\lambda}$ — (7).

i.e. a zone plate has multiple focal length.

Imp

<u>Convex lens.</u>	<u>Zone plate</u>
i) Focal length is given as $\frac{1}{f} = (n-1)\left\{\frac{1}{R_1} + \frac{1}{R_2}\right\}$.	Focal length is given as. $\frac{1}{f_n} = \frac{n\lambda}{r_n^2}$
ii) As $n \propto \frac{1}{\lambda}$. $\Rightarrow f \propto \lambda$.	In this case the focal length is inversely proportional to λ. i.e $f_n \propto \frac{1}{\lambda}$
iii) Light takes the same time to go from the object to the position of the image, while passing through the lens.	In case of zone plate the light waves from any transparent zone reaches the point I one period later than the disturbances from the next inner zone.
iv) lens produces only one image at a fixed distance of the object	It produces a number of images i.e. it has multiple focii

Diffraction at st. Edge. (Determination of wavelength of light from the study of fringes) #5

Consider a st edge C illuminated by a narrow slit S parallel to each other and ⊥ to the plane of the paper. We have to determine the illumination on the screen TR ⊥ to the plane of the paper. Draw a st line SCO s.t SO⊥ screen RT. Clearly this line defines the limits of the geometrical shadow.

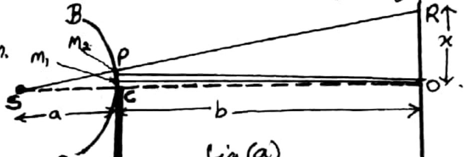


fig (a)

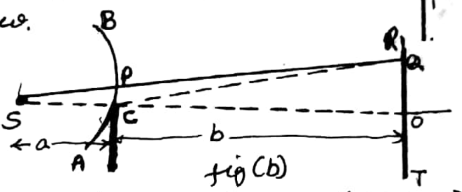


fig (b)

As per geometrical optics the illumination should begin abruptly at O but in practice we observe fringes (i.e dark and bright bands of unequal width and of varying intensity). This intensity goes on decreasing as we move into the geometrical shadow (i.e OT region)
As discussed previously to calcuate the intensity at pt O (or any other pt) we ~~take~~ make use of Fresnel's Half period zones and the net intensity at the pt. being dependent upon the number of half period zones. fig ⓪ represents the half period zones as CM_1, M_2 ......

Similarly consider any pt. Q at distance x from O. Join QS meeting the wave front at P. Now taking P as pole we construct Fresnels half period zones. The net effect at Q depends upon the number of half period elements contained in PC plus the effect of the upper half of the wave i.e PB. Now at O the upper half of the wavefront is effective, thus it has half of the displacement (or $\frac{1}{4}$ of intensity) which would have been there if the whole wave-front were effective. As we move from O to T (inside the geometrical shadow) The wave front moves or the pole P moves from P to A which in turn leads to the interception of the first, second, third etc. half period elements, thus the intensity decreases gradually. Also as we move from O towards R the first, second, third etc. half period elements get exposed. Thus at pt. Q. the resultant illumination is due to these half period zones in PC plus those in PB. Also the amplitude at Q would be maximum or minimum accordingly as CP contains 'odd' or 'even' number of half period elements. At greater distances from O towards R we observe a uniform illumination (as no half period zone is obstructed)

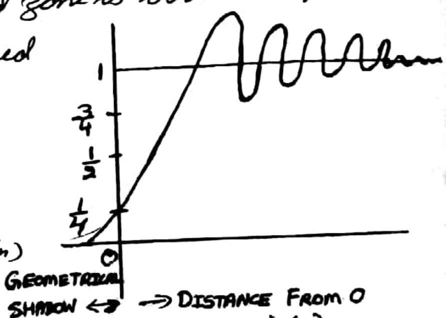


fig (c).

__Mathematically__ The numbe of half period elements contained in CP depends upon the path difference CQ−PQ.

As per figure $SC = a$; $CO = b \Rightarrow CQ = \sqrt{b^2+x^2}$

$$= b\left(1+\frac{x^2}{b^2}\right)^{1/2} = b + \frac{1}{2}\frac{x^2}{b}$$

Since x is very small as compared to b, thus neglecting higher terms (powers) of b (here we have applied Binomial expansion)

Similarly $SQ = \left((a+b)^2 + x^2\right)^{1/2}$

$$= (a+b) + \frac{x^2}{2(a+b)}$$

hence $PQ = SQ - SP =$

$$\Rightarrow PQ = (a+b) + \frac{x^2}{2(a+b)} - a$$

$$= b + \frac{x^2}{2(a+b)}$$

$\therefore$ Path difference $= CQ - PQ$

$$= b + \frac{x^2}{2b} - b - \frac{x^2}{2(a+b)}$$

$$= \frac{ax^2}{2b(a+b)}$$

For max. displacement

$$\frac{ax^2}{2b(a+b)} = (2n+1)\frac{\lambda}{2}.$$

$$\Rightarrow x = \sqrt{\frac{b(a+b)(2n+1)\lambda}{a}}$$

For minimum displacement

$$\frac{ax^2}{2b(a+b)} = n\lambda.$$

$$\Rightarrow x = \sqrt{\frac{b(a+b)2n\lambda}{a}}$$

clearly as $x \propto \sqrt{n}$ and $x \propto \sqrt{2n+1}$ for dark and bright fringes thus the fringe width does on decreasing as we move away from O. i.e the bands produced are of unequal breadth. fig (c).

__Determination of Wavelength__

A st. edge say a sharp razor blade is set with its edge ∥ to the slit on one of the stands on an optical bench. The slit is illuminated with a monochromatic light. Micrometer eyepiece is used for observing the fringes. The position of the first maximum and the one say nth most distant & clearly visible is noted then:

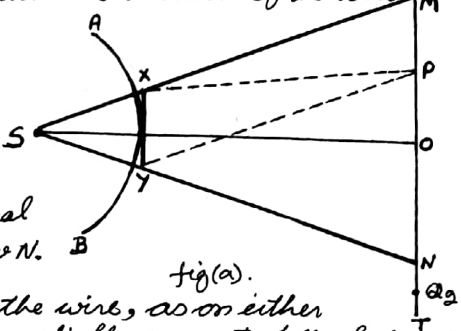
$$x_1 = \sqrt{\frac{b(a+b)\lambda}{a}} \ ; \ n = 0.$$

$$\& \ x_n = \sqrt{\frac{b(a+b)(2n+1)\lambda}{a}}$$

$$\therefore x_n - x_1 = \sqrt{\frac{b(a+b)\lambda}{a}}\left[\sqrt{2n+1} - 1\right]$$

The distances b & a are measured. where
$b \rightarrow$ dist of st edge & eye piece
$a \rightarrow$ dist of source & st. edge.
knowing $(x_n - x_1)$, a & b we can find λ the wavelength of light.

Diffraction of monochromatic light by a thin wire. How does diameter of wire effects the diffraction patterns. How is it employed to measure the thickness of the wire #6



fig(a).

Let S represent a narrow slit placed parallel to the wire of thickness XY and ⊥ to the plane of paper.
As per figure MN is the geometrical shadow on the screen RT.
Now effect at pts Q_1 and Q_2 outside MN is same as that of st. edge at X & Y respectively i.e. unequal bands would be observed above M and also below N.
These bands are independent of the thickness of the wire, as on either side the effect of the other half of the wave is negligible as most of the half period zones are cut off due to the finite width of the wire.
Now within the geometrical shadow the interference fringes are observed.
Also effect of AX is entirely due to a few half period zones at X or X can be considered as a small luminous source at X. Similar is the case of YB.
Further the effect at any point P within MN depends upon the path difference (PY-PX). Also P is bright if $(PY-PX)=n\lambda$ and dark if $(PY-PX)=(2n+1)\lambda/2$
These are equal in width and the fringe width is given as $\beta=\frac{D}{d}\lambda$
where $D\rightarrow$ Distance of screen (RT) from obstacle (XY) & $d\rightarrow$ thickness of obstacle i.e. the diameter of the wire.

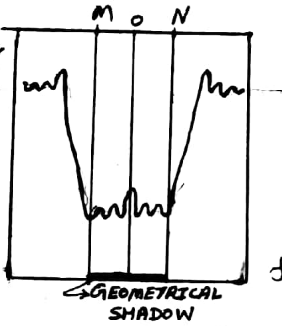


fig (b)

The centre O is bright as the waves from X & Y meet at it in phase. The variation of the fringes as produced within the geometrical shadow of a thin wire and diffraction bands outside are as shown in fig (b)
As the diameter of the wire is increased the fringe width decreases and as the thickness of the wire is sufficiently large the interference fringes disappear and only those outside the geometrical shadow exist as shown in fig (c).

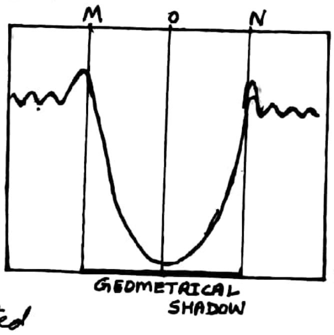
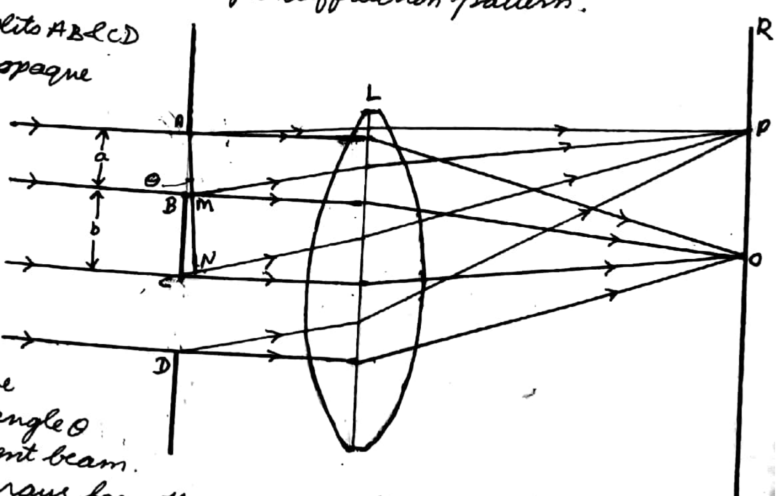


fig (c)

Diameter of the wire To determine it the mean fringe width is observed by the micrometer eye-piece focused within the geometrical shadow. This is done by measuring the shift of the cross-wire for a known number of fringes. Thereafter 'd' can be calcuated from the formulae $\beta=\frac{D}{d}\lambda$

FRAUNHOFER DIFFRACTION

Theory and Features of Double Slit Fraunhofer diffraction pattern.



Consider two parallel rectangular slits AB & CD of equal width 'a' seperated by opaque distance 'b' placed ⊥ to the plane of the paper. S.t distance between two corresponding pts. of the two slits is (a+b)
Consider a plane wavefront of a mono chromatic light of wavelength λ be incident on the slits. Let each slit diffracts the beam in a direction making angle θ with the direction of the incident beam.
Path difference between two rays from the corresponding points A & C of the two slits are

$$CN=(a+b)\sin\theta.$$

If 2β is the phase difference, then

$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \Rightarrow \beta = \frac{\pi}{\lambda}(a+b)\sin\theta$ (i)

If the two slits are considered as two small monochromatic sources then light from them is focused on the screen TR by means of a lens L. Thus patterns are due to interference between lights from two corresponding pts. of the source (slits).

If 2α is the phase difference between the extreme rays from the first slit i.e. from slit AB, then

$2\alpha = \frac{2\pi}{\lambda} a\sin\theta \Rightarrow \alpha = \frac{\pi}{\lambda} a\sin\theta$ —— (ii)

Now resultant displacement y_1 due to rays from the first slit is given by

$y_1 = A\sin\omega t$ —(iii) where $A_\theta = A_0 \frac{\sin\alpha}{\alpha}$

Similarly

$y_2 = A\sin(\omega t + 2\beta)$ —— (iv).

Thus resultant displacement

$$Y = y_1 + y_2 = A\sin\omega t + A\sin(\omega t + 2\beta)$$
$$= 2A\cos\beta\,\sin(\omega t + \beta) \text{ —— (v)}$$

Also amplitude of resultant Y is

$$A_R = 2A\cos\beta = 2A_0\frac{\sin\alpha}{\alpha}\cos\beta.$$

also Resultant intensity is proportional to it

i.e. $I_R \propto 4A_0^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$

Thus the intensity (I_R) depends upon

i) $A_0^2\frac{\sin^2\alpha}{\alpha^2}$ → diffraction pattern due to single slit

ii) $\cos^2\beta$ → system of interference fringes due to wavelets from the corresponding pts. of the two slits.

Diffraction patterns have greater dispersion & Interference patterns have lesser dispersion.

(A) Nature of Diffraction Pattern:-

$\frac{A_0^2\sin^2\alpha}{\alpha^2}$ → diffraction pattern due to single slit.

For "Central Maximum"

$\alpha = 0$ as $\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha} = 1$.

Also position of minima is

$\alpha = \pm n\pi$ or $a\sin\theta = \pm n\lambda$.

Thus for $n = 1, 2, 3 \ldots\ldots$ we get.

$\sin\theta_1 = \pm\frac{\lambda}{a}$; $\sin\theta_2 = \pm\frac{2\lambda}{a}$; $\sin\theta_3 = \pm\frac{3\lambda}{a}$

For secondary maxima

$\alpha = \pm\frac{3\pi}{2}$; $\pm\frac{5\pi}{2}$; $\pm\frac{7\pi}{2}$ .....

or $\sin\theta_1 = \pm\frac{3\lambda}{a}$; $\sin\theta_2 = \pm\frac{5\lambda}{a}$; $\sin\theta_3 = \pm\frac{7\pi}{a}$ ....

(B) Nature of Interference patterns

Consider factor $\cos^2\beta$. From it we have maximum intensity when

$\cos^2\beta = 1$ or $\beta = \pm n\pi$ where $n = 0, 1, 2 \ldots$ #7

$\Rightarrow \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$

$\Rightarrow (a+b)\sin\theta = \pm n\lambda$ —— (vi).

Thus for maxima positions we have

$\sin\theta_1 = \pm\frac{\lambda}{a+b}$; $\sin\theta_2 = \pm\frac{2\lambda}{a+b}$; $\sin\theta_3 = \frac{3\lambda}{a+b}$

clearly the angular separation

$\sin\theta_2 - \sin\theta_1 = \sin\theta_3 - \sin\theta_2 = \pm\frac{\lambda}{a+b}$ —(vii)

Thus the interference "Central Maximum" is given when $\theta = 0$ in the direction of incident rays. This is also the direction of 'central maximum' of the diffraction pattern.

For minima $\cos^2\beta = 0$ or $\beta = \pm(2n+1)\pi/2$

$\therefore \frac{\pi}{\lambda}(a+b)\sin\theta = \frac{(2n+1)\pi}{2}$

$\Rightarrow (a+b)\sin\theta = \pm(2n+1)\lambda/2$.

$n = 1, 2, 3 \ldots\ldots$

Thus $\sin\theta_1 = \pm\frac{3\lambda}{2(a+b)}$; $\sin\theta_2 = \pm\frac{5\lambda}{2(a+b)}$ ....

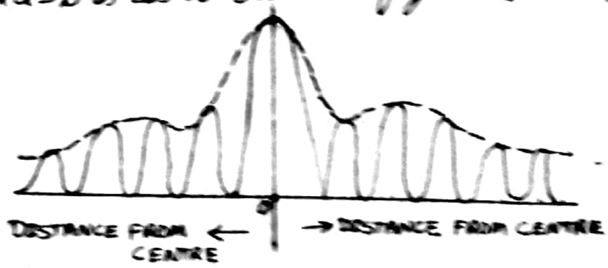
clearly the angular separation between two consecutive minima is

$\sin\theta_2 - \sin\theta_1 = \pm\frac{\lambda}{a+b}$ —— (viii).

Also from eqns (vii) & (viii) the angular separation between two consecutive minima & two consecutive maxima is equal to $\frac{\lambda}{(a+b)}$.

Conclusion The minimum intensity occurs along directions for which the path diff. between two corresponding points of the slits is an odd multiple of $\lambda/2$ and maximum intensity occurs for path difference being even multiple of $\lambda/2$.

For $\beta = 0$, the intensity of central maximum $= 4I_0$ where $\alpha = 0$. The intensity diagram corresponding to $2a = b$ is as shown in figure below.



Missing orders

For interference maxima $(a+b)\sin\theta = n\lambda$ —(ix)

" diffraction minima $a\sin\theta = p\lambda$ —(x)

where $n, p \in I$

If values of a, b are such that both are satisfied for the same value of θ i.e. interference maxima falls on diffraction minima thereby a spectrum order gets missed.

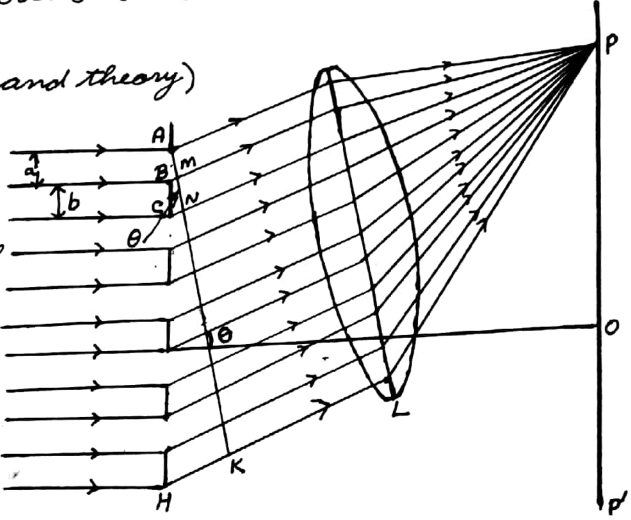
dividing we get $\frac{a+b}{a} = \frac{n}{p}$.

If $a = b$ then $\frac{n}{p} = 2$ or $n = 2p$.

Thus if $p = 1, 2, 3 \ldots$ then $n = 2, 4, 6 \ldots\ldots$ thus 2, 4, 6 th orders interference maxima will be missing in the diffraction pattern. In this case three interference maxima are in the central diffraction maximum.

Imp If $b = 0$, the two two slits join and all interference maxima would be missing and the diffraction pattern will be the same as due to a single slit of width $2a$.

Plane Transmission Grating (Construction and theory)



It is an arrangement equivalent in action to a large number of parallel and equidistant slits of the same width. It is constructed by ruling equidistant parallel lines on a transparent material such as glass plate by means of a fine dimond point worked with a ruling engine. The rulings act as opaque wires called as **opacties**. Light passes through the spaces in between the lines called as the **transparencies**. The number of lines on a plane transmission grating is of the order of 15000 lines per inch

THEORY let $ABC \ldots H$ be the section of a plane transmission grating $\perp$ to plane of paper. let $a \rightarrow$ length of opaque portion (BC etc) and $b \rightarrow$ length of transparent sections (AB etc)

Here $(a+b) \rightarrow$ **grating element or grating constant**

Consider a parallel beam of monochromatic light (plane wave) of wavelength λ be incident normally on the grating surface. Most of the light goes st. and is brought to focus by the lens L at O giving rise to **central maximum**. Now diffraction also takes place as width of slits is comparable to that of the incident ~~so~~ monochromatic light. Consider light from two corresponding pts A & C in a direction making an angle θ with the normal to the slit. In order to find the path difference Draw $AK \perp$ to direction of diffracted rays, i.e $\angle HAK = \theta$. Thus the path CN is given by.

$$CN = AC \sin\theta = (a+b)\sin\theta.$$

Now any pt P will have maximum intensity if this path difference is an integral multiple of λ

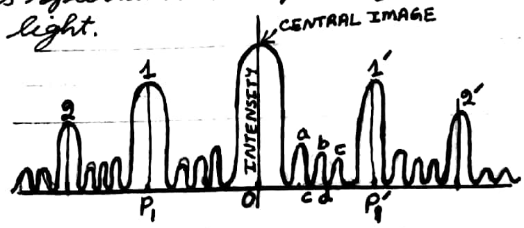
i.e. $(a+b)\sin\theta_n = n\lambda$ where $\theta_n \rightarrow$ direction of principle maximums

For $n=0$ we get central maximum at O

For $n=1 \Rightarrow (a+b)\sin\theta_1 = \lambda$ direction of 1st maximum on either side of O i.e P & P'

For $n=2 \Rightarrow (a+b)\sin\theta_2 = 2\lambda$ direction of 2nd order maxima

Imp For white light the diffraction pattern on the screen will consist of central bright maximum surrounded on both sides spectrum corresponding to different wavelengths of the constituents of white light.



The intensity distribution on the screen is as shown in figure. At pt. O the intensity is maximum and corresponds to the intensity of central maximum. At P_1 and P_1' the we have first order maximum. Inbetween central maxima and first order maximum we have a number of secondary maxima and secondary minima. The intensity of these intervening secondary maxima and minima ~~so~~ is so low that these not generally observed and thus there is a uniform darkness between any two principal maxima.

Missing orders of a diffraction grating

Now nth principal maxima in a plane diffraction grating is given by

$$(a+b)\sin\theta_n = n\lambda \quad \text{———} \quad (1)$$

If the values of a and θ_n are such that

$$a \sin\theta_n = \lambda \quad \text{——— (2)}$$

then the effect of wavefronts from any particular slit will be zero, as the slit is divided into two halves due to which the secondary waves from the corresponding points of the two halves will be $\lambda/2$, thus the resulting intensity is zero. If conditions (1) & (2) are simultaneously obeyed then the spectrum will be abscent i.e. dividing eqn (1) by (2) we get.

$$\frac{(a+b)}{a} = n \quad \text{——— (3)}$$

Thus values of missing spectrum are obtained by subsituting the values of n.

Imp eg. if $a=b$. then $\frac{a+a}{a} = n \Rightarrow n=2$. Thus the second order spectrum will be abscent.

M.Imp Now principal maxima are given by

$$(a+b)\sin\theta_n = n\lambda$$

For first order principal maxima $(a+b)\sin\theta_1 = \lambda \Rightarrow \sin\theta_1 = \frac{\lambda}{a+b}$

If $(a+b)=\lambda$ then $\sin\theta_1 = \frac{\lambda}{a+b} = \frac{\lambda}{\lambda} = 1$ or $\theta_1 = 90^\circ$

Again if $(a+b)<1 \Rightarrow \sin\theta > 1$ which is not possible. Thus if $a+b=\lambda$ or $(a+b)<\lambda$ first order spectrum will not be possible. Similarly 2nd, 3rd principal maxima will be also not possible.

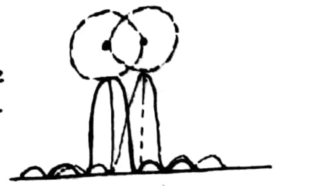
"Do expt. yourself to determine the wavelength λ of a monochromatic source of light"

Magnifying Power:- It is defined as the ratio of the angle subtended by the image at the eye as seen through the instrument to the angle subtended by the object at the centre of the unaided eye when the image and the object both lie at the same distance from the eye. This distance for a microscope is the distance of distinct vision and for a telescope it is infinity.

Resolving Power:- It is defined as the resiprocal of the smallest angle subtended at the objective by two point objects which can just be distinguished as separate.

Magnifying Power.	Resolving Power.
1. It is the ability of the telescope to show a magnified view of the object.	It is the ability of the telescope to show the amount of detail in the object.
2. It gives no idea about the size of central maximum.	It not only gives the idea about the distance between the centres of the two maxima, but also gives us the idea about its width
3. It is independent of the wavelength of light	Resolving power increases as the wavelength of light decreases.

Rayleigh's Criterion of resolution



Two close lines in the spectrum are said to be just resolved when the central maximum of the diffraction pattern of one falls on the first minimum of the diffraction pattern of the other i.e. the least angular separation between the principal maxima of the diffraction pattern of the two spectral lines in a given diffraction order should be equal to half the angular width of either principal maximum. The resolving power in that case is $\lambda/d\lambda$ where λ is the wavelength of a line in the spectrum and $d\lambda$ is the least difference in wavelength that can just be seen as separate thus

$$\text{Resolving power} = \frac{\lambda}{d\lambda}.$$

"Angular dispersive" and the "resolving power" of a grating.

Angular dispersive power:- It is defined as the change in the angle of diffraction corresponding to a unit change in the wavelength.

The expression for the principal maxima in the nth order for a diffraction grating is $(a+b)\sin\theta = n\lambda$ —— ①

This expression shows that θ varies with wavelength, thus for wavelength change from λ to $(\lambda+d\lambda)$, the angle of diffraction changes from θ to $(\theta+d\theta)$

Then the ratio $\frac{d\theta}{d\lambda}$ is called as the angular "dispersive power"

Expression diff. ~~we get~~ $(a+b)\sin\theta = n\lambda$ we get

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

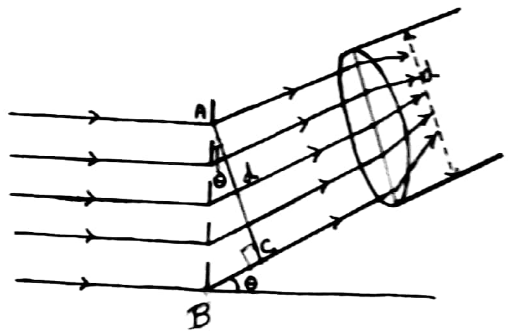
$$\therefore \text{dispersive power } \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \quad \text{—— ②}$$

Clearly from this =n. The dispersive power of a grating increases with i) The order of the spectrum and ii) The decrease of the grating element i.e the increase in the number of lines per cm.

Resolving Power:- It is defined as the ratio of the wavelength of a line in the spectrum to the least difference in the wavelength of the next line that can just be seen a separate. Moreover two lines of wavelengths λ and $(\lambda+d\lambda)$ are said to be just resolved in a certain order of the grating spectrum if the primary maximum of one falls on the first secondary minimum of the other.

Expression

$$(a+b)\sin\theta = n\lambda \quad \text{—— ③}$$



If λ goes to $(\lambda+d\lambda)$ then θ goes to $(\theta+d\theta)$ and relation between $d\lambda$ and $d\theta$ can be obtained by diff. =n③ as.

$$(a+b)\cos\theta \cdot d\theta = n\, d\lambda$$

$$\Rightarrow d\theta = \frac{n\, d\lambda}{(a+b)\cos\theta}$$

The light after suffering diffraction at the grating enters the objective of the telescope in a spectrometer. If d is the diameter of the telescope objective, the smallest angle $d\theta'$, that it is able to resolve for a light of wavelength λ is given by $d\theta' = \frac{\lambda}{d}$

Thus the two principal maxima in the certain order of the grating spectrum corresponding to wavelengths λ and $(\lambda+d\lambda)$ having an angular separation $d\theta$ will be just resolved by the telescope if

$$d\theta' = d\theta \quad \text{or} \quad \frac{n\cdot d\lambda}{(a+b)\cos\theta} = \frac{\lambda}{d}$$

$$\Rightarrow \frac{\lambda}{d\lambda} = \frac{n\cdot d}{(a+b)\cos\theta} \quad \text{—— ④}$$

If the total width of the grating is AB and it contains N lines, then

$AB = N(a+b)$ also as $AC = d = AB\cos\theta = N(a+b)\cos\theta$

Substituting this value in =n(IV) we get

$$\frac{\lambda}{d\lambda} = \frac{n\cdot N(a+b)\cos\theta}{(a+b)\cos\theta} = nN$$

Thus in order to use the full resolving power of the grating in a certain order of the spectrum the aperture of the telescope objective should ~~be~~ at least be equal to $AB\cos\theta$ where θ is the angle of diffraction corresponding to that order of the spectrum.

Scanned with CamScanner

m.imp 'A Grating having higher dispersive power' than another does not necessarily has a higher resolving power.

The angular dispersive power is given as $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{nN'}{\cos\theta}$ —— ①

where $N' \rightarrow$ no. of lines per cm. of the grating.

Clearly for a given order spectrum the angular dispersive power depends upon the no. of lines per cm. If N' is increased, the dispersive power is also increased. The resolving power is given as $\frac{\lambda}{d\lambda} = nN$ —— ②

where $N \rightarrow$ total no. of lines on the effective width of the grating. Substituting the value of n in ② we get

$$\frac{\lambda}{d\lambda} = \frac{N(a+b)\sin\theta}{\lambda} = \frac{\omega\sin\theta}{\lambda}$$

where $\omega \rightarrow$ width of the ruled surface thus increase of no. of lines does not increase the resolving power of the grating.

Hence a grating with higher dispersive power i.e. increase in the no. of lines per cm. does not ~~necessarily~~ increase the resolving power

Imp

Prism spectrum	Grating Spectrum
1. There is only one spectrum	There are a no. of spectrum on either side of the central maximum.
2. Deviations produced depend upon the angle of prism, refractive index of the material of prism & λ of light (it is more for violet than for red as $\lambda_v > \lambda_r$)	Deviation is directly proportional to the λ of light used and inversely proportional to the grating element. It is independent of the material of the grating
3. Dispersive power is more for small λ i.e. violet is less intense than the red portion	For grating dispersive power is given as $\frac{n}{(a+b)\cos\theta}$ i.e. (i) angular dispersion is almost uniform for all lines of a particular order (ii) Seperation in the second order is double of what it is in the first order (iii) the angular separation increases as the grating element decreases
4. The spectra produced by any two prisms are neither similar nor regular.	The spectrum of two gratings are exactly similar for a given source Only the length is directly proportional to the no. of lines per cm of the grating
5. Resolving power of prism is small & is dependent upon the length of the base of prism and the wavelength of light	The resolving power is high and given by Nn where $N \rightarrow$ no. of lines in effective part & $n \rightarrow$ order of the spectrum.

Problems.

#1. Deduce the missing orders for a double slit, Fraunhofer diffraction pattern if the slit widths are 0.16 mm and they are 0.8 mm apart

Sol width of slit $a = 0.16mm = 0.016cm$

space between slits $b = 0.8mm = 0.08cms.$

now $\frac{n}{p} = \frac{a+b}{a} = \frac{0.016+0.08}{0.016} = 6.$

Thus for $p = 1, 2, 3 \ldots$ the values of n are $n = 6, 12, 18 \ldots$ i.e orders 6, 12, 18 etc of interference maxima will be missing from the diffraction pattern.

#2 A plane beam of Na light is allowed to be normally incident on a plane grating having 4250 lines/cm. The second order spectral line is observed at 30°. calculate λ

Sol. Grating element $(a+b) = \frac{1}{N} = \frac{1}{4250}$

also. $n\lambda = (a+b)\sin\theta_n$

for 2nd order as $n=2$ hence.

$$2\lambda = (a+b)\sin\theta_2$$

$$\text{or } \lambda = \frac{(a+b)\sin\theta_2}{2} = \frac{1 \times \sin 30^\circ}{4250 \times 2}$$

$$\lambda = \frac{0.5}{2 \times 4250} = 5882 \times 10^{-8} cm = 5882^\circ A$$

#3 How many lines per cm are there in a grating which gives a diffraction of 30° in the first order for a mono-chromatic light having $\lambda = 6 \times 10^{-5} cm.$

Sol $\lambda = 6 \times 10^{-5} cm$; $\theta_1 = 30^\circ$

$(a+b)$ grating element $= \frac{1}{N}$

also $n\lambda = (a+b)\sin\theta_n$

for Ist order $n=1$ hence.

$$1 \times 6 \times 10^{-5} = \frac{1}{N} \times \sin 30^\circ.$$

$$\frac{1}{N} = \frac{6 \times 10^{-5}}{0.5}$$

$$\text{or } N = \text{lines/cm} = \frac{0.5}{6 \times 10^{-5}} = 8333.3$$

#4 Show that 4th or higher order of light of wavelength 5890°A is missing for a grating having 5000 lines/cm.

Sol $\lambda = 5890 \times 10^{-8} cm$; $N = 5000$ lines/cm.

$$\therefore (a+b) = \frac{1}{N} = \frac{1}{5000} = 0.0002 cm.$$

let $n \rightarrow$ max. no. of orders possible. also 90° is max. angle of diffraction.

thus $n = \frac{a+b}{\lambda} = \frac{0.0002}{5890 \times 10^{-8}} = 3.4$ approx

or $n = 3$ i.e only 3rd order spectrum is possible while 4th & higher orders is missing.

$\boxed{\therefore \text{dispersive Power } \frac{d\theta}{d\lambda}}$

#5 Calculate the angular dispersion in degrees per angstrom for a diffraction grating having 14,438 lines/inch when used in the third order at 4200°A.

Sol $\lambda = 4200 \times 10^{-8} cm$; $n = 3$; $N = 14438$

Grating Const. $(a+b) = \frac{2.54}{14438}$

from grating formula $n\lambda = (a+b)\sin\theta_n$

$$\sin\theta_3 = \frac{3 \times 4200 \times 10^{-8} \times 14438}{2.54}$$

$$\sin\theta_3 = 0.7162$$

Now angular dispersion $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta_n}$

here. $\cos\theta_n = \sqrt{1-\sin^2\theta_n}$

$$\cos\theta_3 = \sqrt{1-\sin^2\theta_3} = 0.6979$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{3 \times 14438}{2.54 \times 0.6979} \simeq 0.0002443 \text{ rad.}$$

$$= \left(\frac{0.0002443}{\pi}\right)^\circ$$

#6 Calculate dispersive Power of a grating in 3rd order given $\lambda = 5000^\circ A$, $N = 4000$ lines/cm. used at normal incidence.

Sol

Dispersive Power $\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta_3}$ $\left[\text{OR } \frac{n}{(a+b)\cos\theta_n}\right]$

$n = 3$; $N' = 4000$ $\cos\theta_3 = ?$

Now $(a+b)\sin\theta_3 = 3\lambda.$

$$\sin\theta_3 = \frac{3\lambda}{a+b} = 3\lambda N' = 3 \times 5000 \times 10^{-8} \times 4000 = 0.6$$

$$\therefore \cos\theta_3 = \sqrt{1-\sin^2\theta_3} = 0.8.$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{3 \times 4000}{0.8} = 15000$$

Imp

#7 Calculate the minimum no. of lines that a grating should have just to resolve the sodium doublet of wavelengths 5890°A and 5896°A in the third order spectrum.

Sol $\lambda_1 = 5890^\circ A$; $\lambda_2 = 5896^\circ A$ $\therefore \lambda_{mean} = 5893^\circ A$

& $d\lambda = 6^\circ A.$

$$\frac{\lambda_m}{d\lambda} = Nn$$

$N \rightarrow$ no of lines a grating should have in order to resolve two wavelengths of order n. (=3 in our case)

$$\therefore N = \frac{\lambda}{d\lambda}\frac{1}{n} \Rightarrow N = \frac{5893}{6 \times 3} = 327.4 = 328 \text{ lines}$$